Local Search

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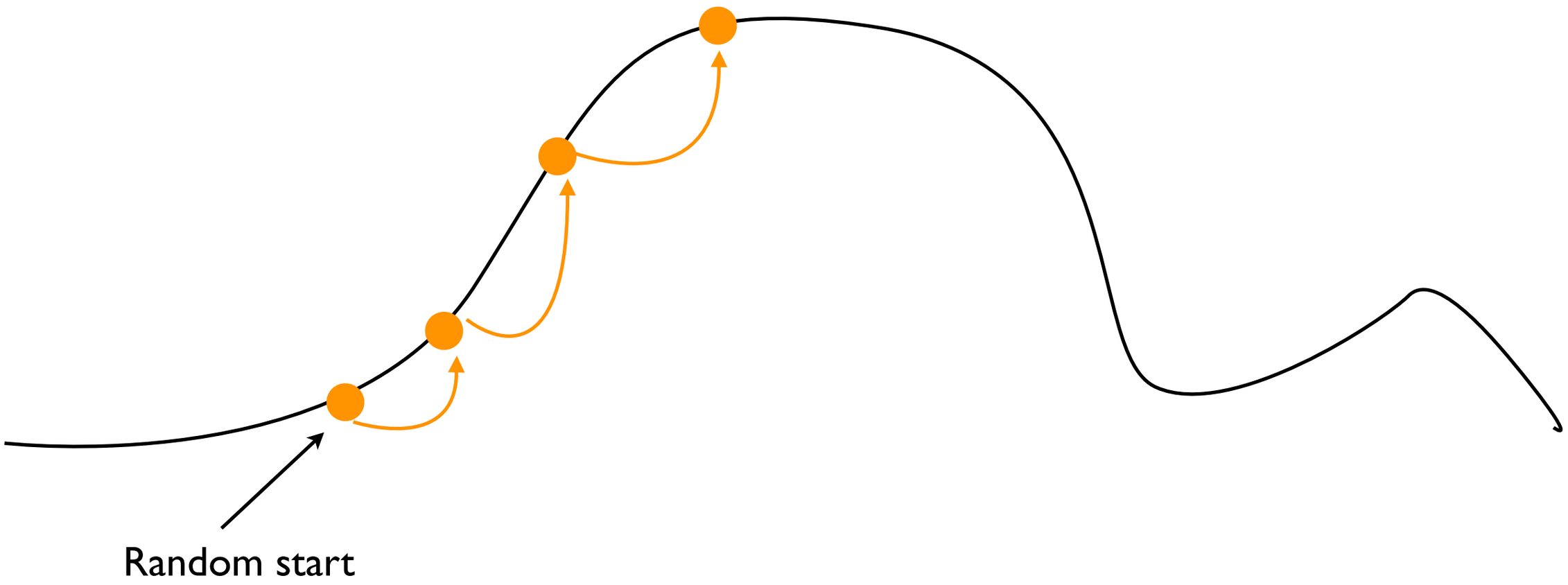
If your problem forms a fitness **landscape**, what is optimisation?

Local Search Loop

- Start with a single, random solution
- Consider the neighbouring solutions
- Move to one of the neighbours if better
- Repeat until no neighbour is better

Local Search



Hill Climbing Algorithm

- This particular variation is also known as the steepest-ascent hill climbing.
- Why? :)
- What other versions are there?

```
HillClimbing()
(1)     climb ← True
(2)     s ← GetRandom()
(3)     while climb
(4)         N ← GetNeighbours(s)
(5)         climb ← False
(6)         foreach n ∈ N
(7)             if Fitness(n) > Fitness(x)
(8)                 climb ← True
(9)                 s ← n
(10)    return s
```

Hill Climbing Algorithm

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Steepest Ascent

```
HillClimbing()
(1)     climb ← True
(2)     s ← GetRandom()
(3)     while climb
(4)         N ← GetNeighbours(s)
(5)         climb ← False
(6)         foreach n ∈ N
(7)             if Fitness(n) > Fitness(x)
(8)                 climb ← True
(9)                 s ← n
(10)                break
(11)    return s
```

First Ascent

Hill Climbing Algorithm

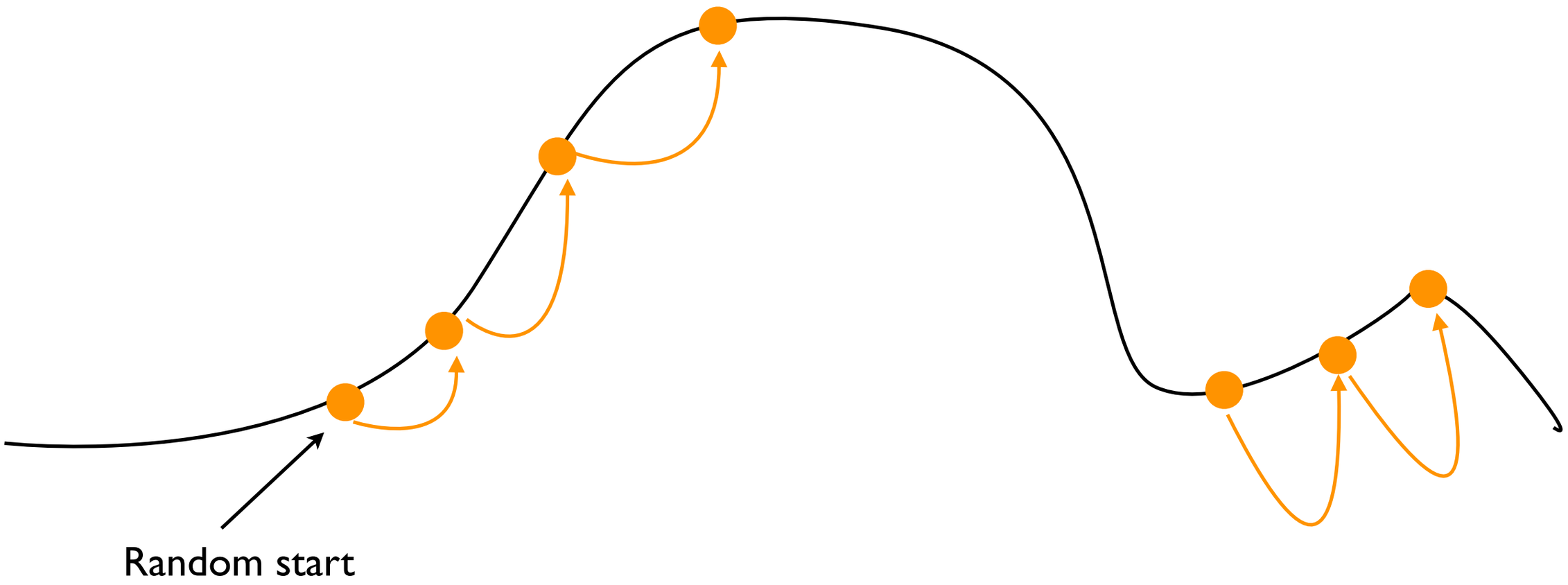
```
HillClimbing()
(1)      climb ← True
(2)      s ← GetRandom()
(3)      while climb
(4)          N ← GetNeighbours(s)
(5)          climb ← False
(6)          foreach n ∈ N
(7)              if Fitness(n) > Fitness(x)
(8)                  climb ← True
(9)                  s ← n
(10)                 break
(11)     return s
```

First Ascent

```
HillClimbing()
(1)      s ← GetRandom()
(2)      while True
(3)          N ← GetNeighbours(s)
(4)          N' ← {n ∈ N|Fitness(n) > Fitness(s)}
(5)          if |N'| > 0
(6)              s ← RandomPick(N')
(7)          else
(8)              break
(9)      return s
```

Random Ascent

Local Search



Ascent Strategy

- Not possible to know which one is better.
- IF the current solution is near a local optimum, slowing down the ascent may (or may not) be better.
- Regardless of strategy, hill climbing monotonically climbs, until it reaches local/global optimum; it never goes down.

Pros/Cons

- Pros:cheap (fewer fitness evaluations compared to GAs), easy to implement, repeated applications can give insights into the landscape, suitable for solutions that need to be built through small incremental changes

- Cons: more likely to get stuck in local optima, unable to escape local optima

Alternating Variable Method (AVM)

- A type of Pattern Search: searches for an input vector that can maximise/minimise a given objective function

- It has two operation modes: exploratory move, and pattern move.

- For each variable:

    - Use exploratory move to decide which direction results in fitter solutions

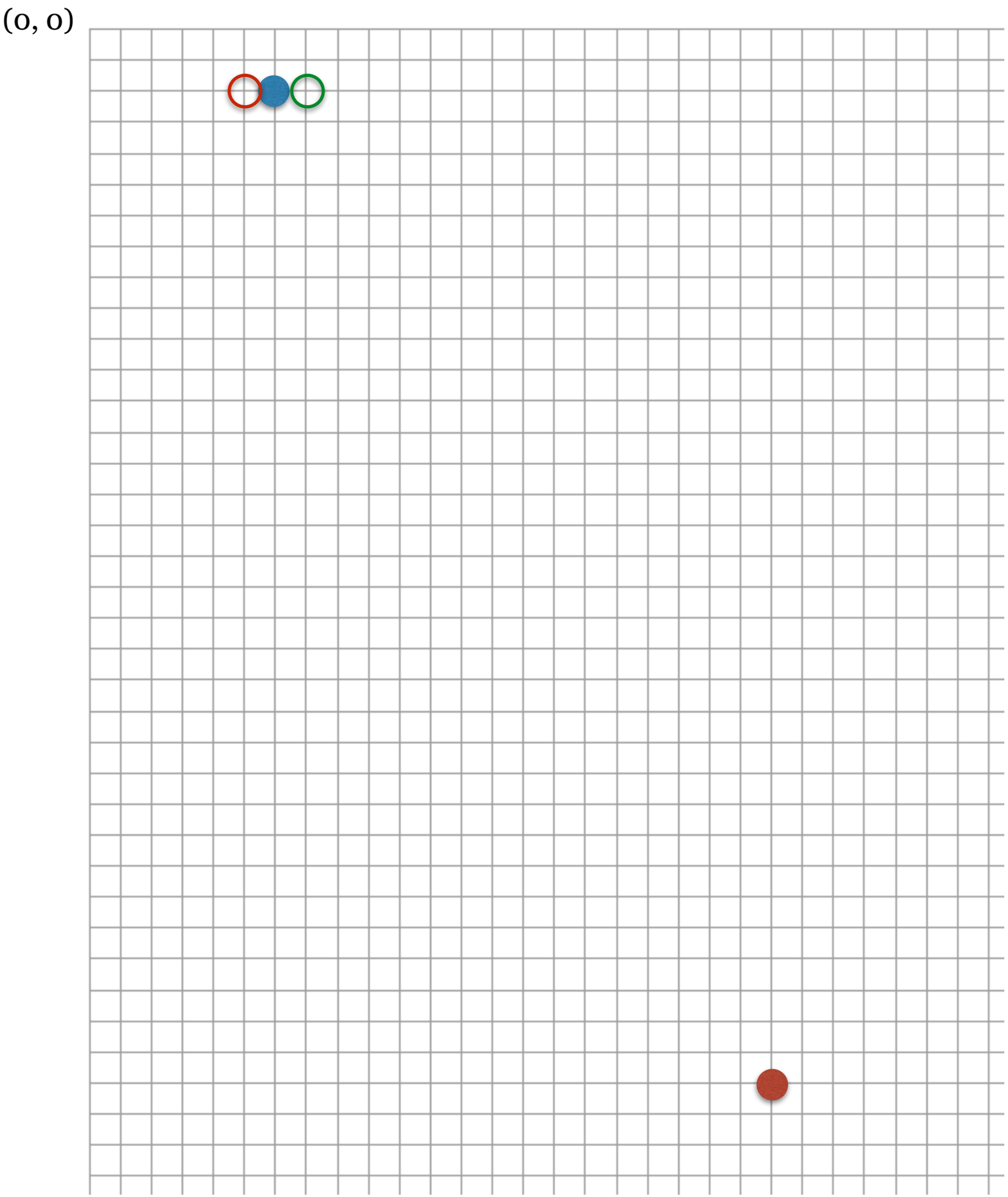
    - Use pattern move to accelerate to that direction

Alternating Variable Method

- Based on the known empirical results, AVM is one of the most effective algorithm for achieving C/C++ structural coverage

  - M. Harman and P. McMinn. A theoretical and empirical analysis of evolutionary testing and hill climbing for structural test data generation. In Proceedings of the International Symposium on Software Testing and Analysis (ISSTA 2007), pages pp. 73–83. ACM Press, July 2007.

  - M. Harman and P. McMinn. A theoretical and empirical study of search based testing: Local, global and hybrid search. IEEE Transactions on Software Engineering, 36(2):226–247, 2010.



AVM: Exploratory Move

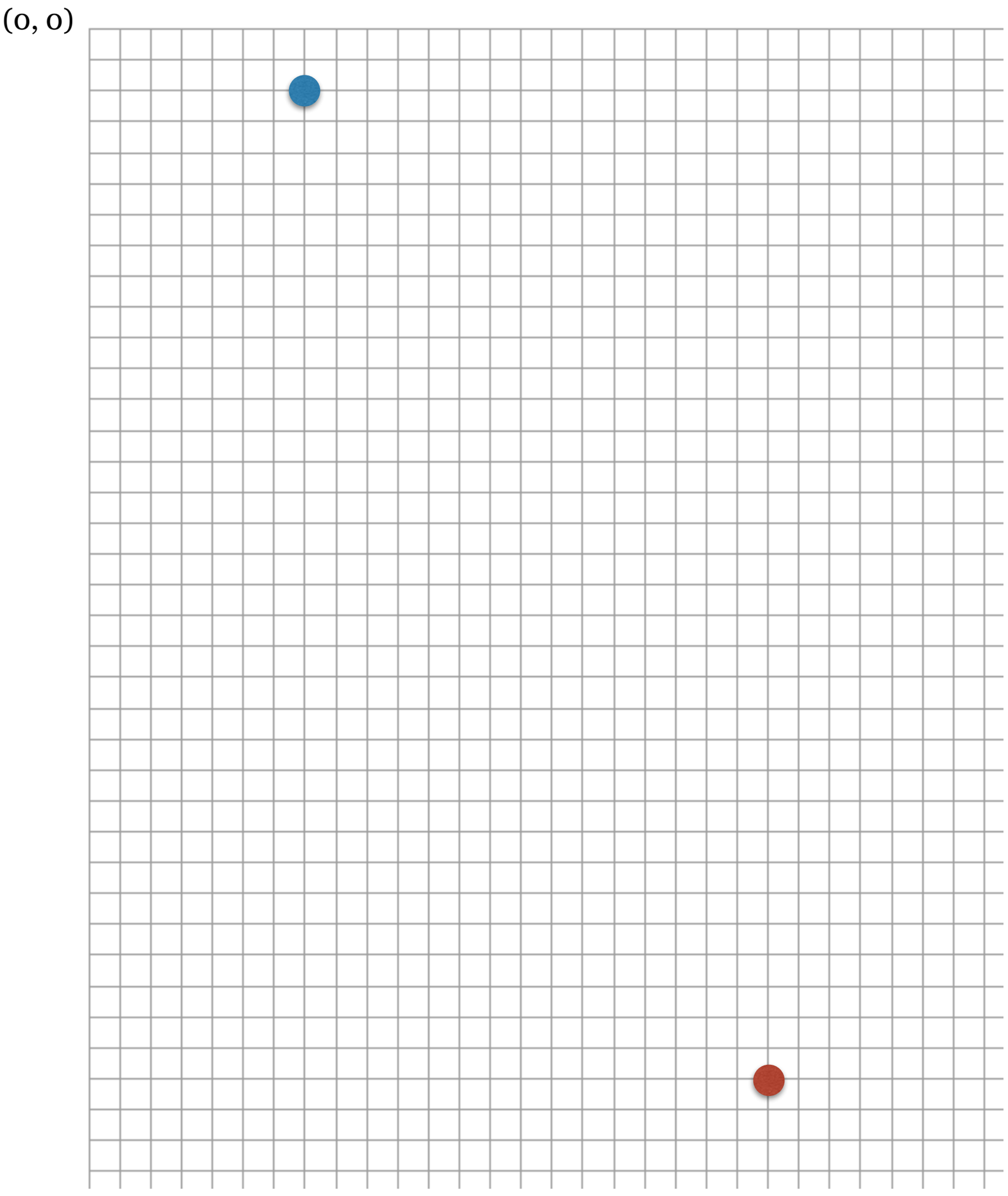
Starting from(6, 2), we want to search for the red dot at (22, 34). We can measure the distance to the goal.

First we try exploratory move for x: make the smallest change, and see which direction results in reduced distance. The initial distance is 35.77.

-1: (5, 2) **Increased** (36.23). X

+1: (7, 2) **Decreased** (35.34) O

Consequently, x needs to be increased at the moment.



AVM: Pattern Move

Now that we decided to increase x, try doubling the difference as long as the distance continues to decrease. At the beginning of the pattern move, x is equal to 7.

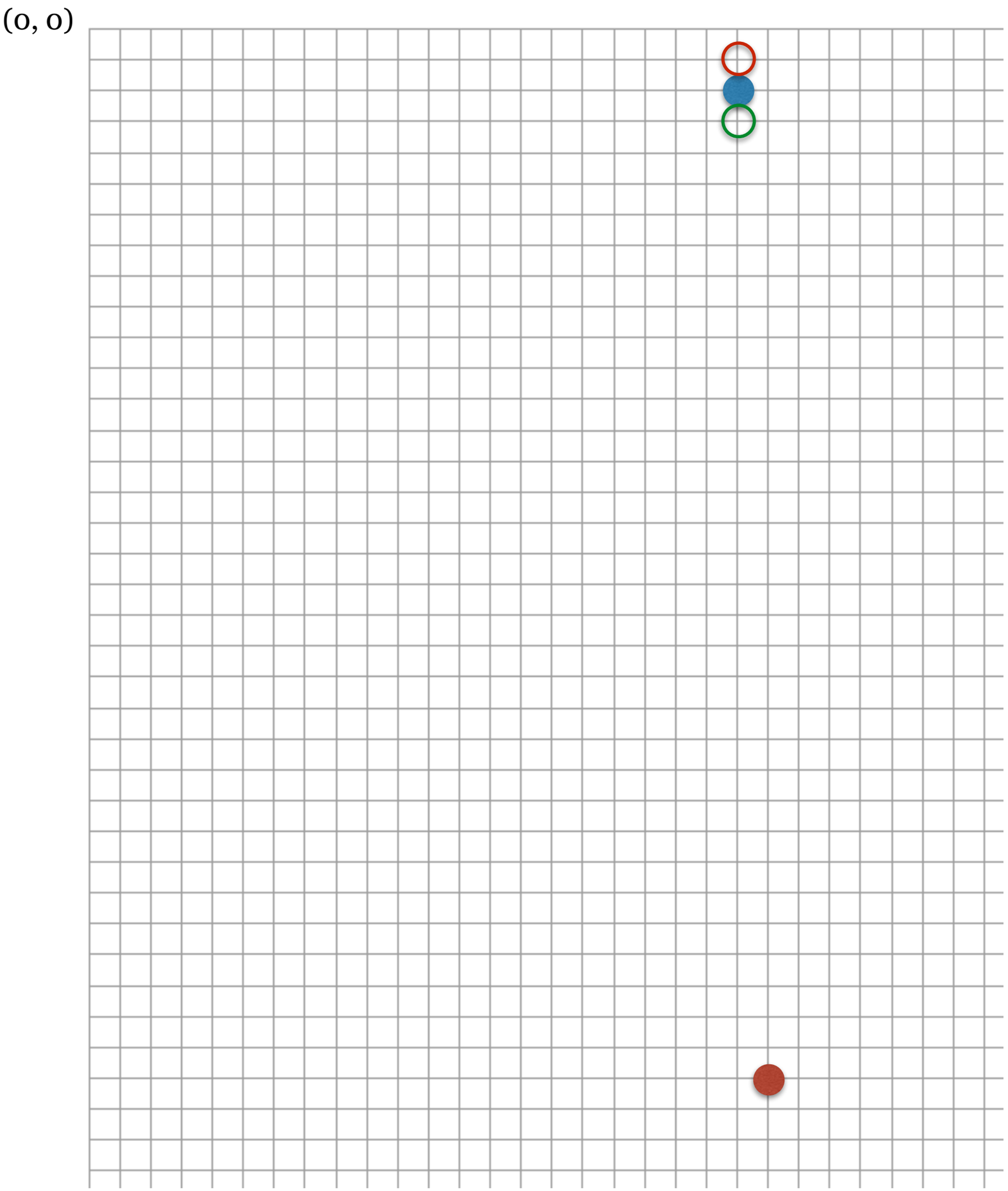
x = 9 (Δx=2): **decrease** (34.53)

x = 13 (Δx=4): **decrease** (33.24)

x = 21 (Δx=8): **decrease** (32.01)

x = 37?(Δx=16): **increase** (35.34)

With increment of 16, the distance starts to grow: this is called overshooting. In this case, we cancel the last pattern move, and start the exploratory move for the next variable, y.



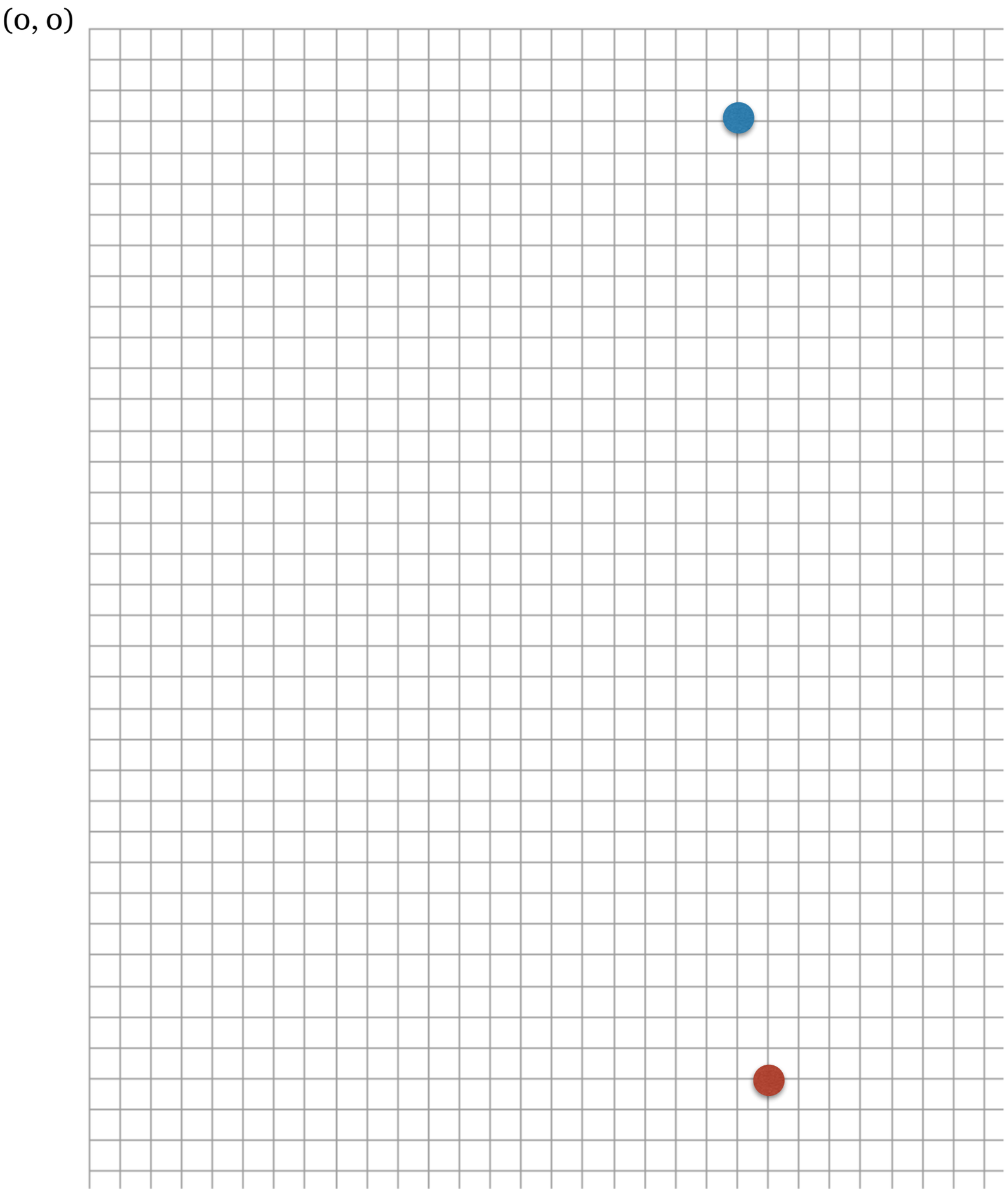
AVM: Exploratory Move

We now change y by 1 and decide the direction. The distance from the last location, (21, 2), is 32.01.

-1: (21, 1) **increase** (33.01). X

+1: (21, 3) **decrease** (31.01) O

So y needs to be increased.



AVM: Pattern Move

We increase the variable y with pattern moves now. Initially y is 3.

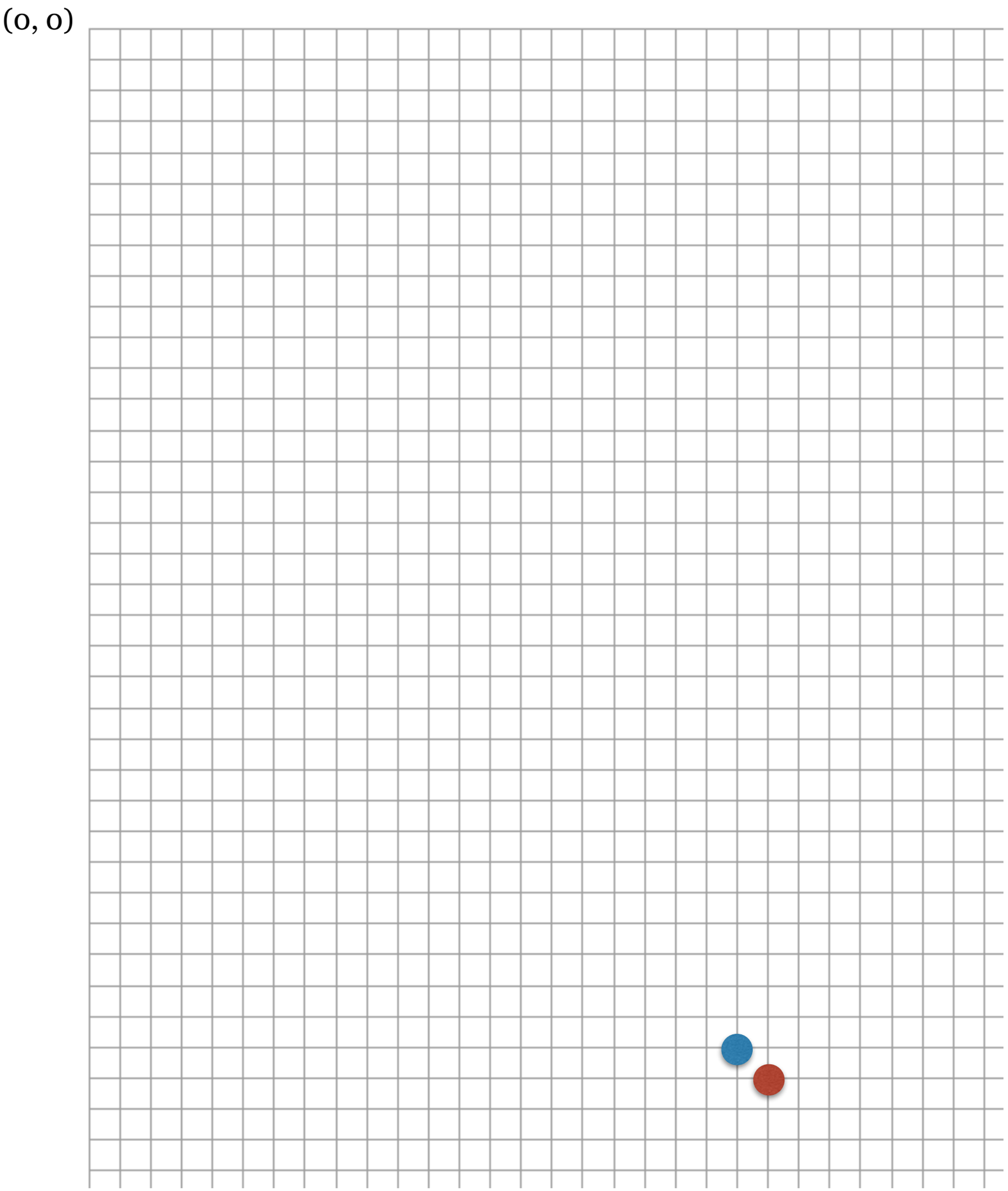
y = 5 (Δy=2): **decrease** (29.01)

y = 9 (Δy=4): **decrease** (25.01)

y = 17 (Δy=8): **decrease** (17.02)

y = 33(Δy=16): **decrease** (1.41)

y = 65(Δy=32): **Overshooting!**



AVM: Exploratory Move

After overshooting of y, we start the exploratory move for x. We decide to increase, but as soon as we try +2, it overshoots. After cancellation of this, we have the correct x.

After one more exploratory move for y, we reach the goal.

Alternating Variable Method

- For a reference implementation and basic applications, see: http://avmframework.org

- P. McMinn and G. M. Kapfhammer. AVMf: An open-source framework and implementation of the alternating variable method. In International Symposium on Search-Based Software Engineering (SSBSE 2016), volume 9962 of Lecture Notes in Computer Science, pages 259–266. Springer, 2016.

Simulated Annealing

- Big question: how do we escape local optima, if we are in one?
    - Thought 1: we never know whether we are climbing a local or a global optimum!
    - Thought 2: assuming that there are more local than global optima, it makes sense to escape.
    - Thought 3: but not always - when we stop, we want to stop near the top of SOME optimum.

Annealing (풀림)

- Keep metal in a very high temperature for a long time, and then slowly cool down: it then becomes more workable.

- At high temperature, atoms are released from internal stress by the energy; during the cool-down, they form new nucleates without any strain, becoming softer.

Simulated Annealing

- Introduce “temperature” into local search: start with a high temperature, and slowly cool down.
- When the temperature is high, the solution (like atom) is unstable and can make random moves (i.e. escapes).
- As the temperature decreases, the energy level gradually gets lower, and escapes become more infrequent.

Simulated Annealing

SimulatedAnnealing()

(1) $s = s_0$
(2) $T \leftarrow T_0$
(3) **for** $k = 0$ **to** n
(4) $s_{new} \leftarrow$ GetRandomNeighbour(s)
(5) **if** $P(\mathrm{F}(s), \mathrm{F}(s_{new}), T) \geq random(0,1)$ **then** $s \leftarrow s_{new}$
(6) $T \leftarrow$ Cool(T)
(7) **return** s

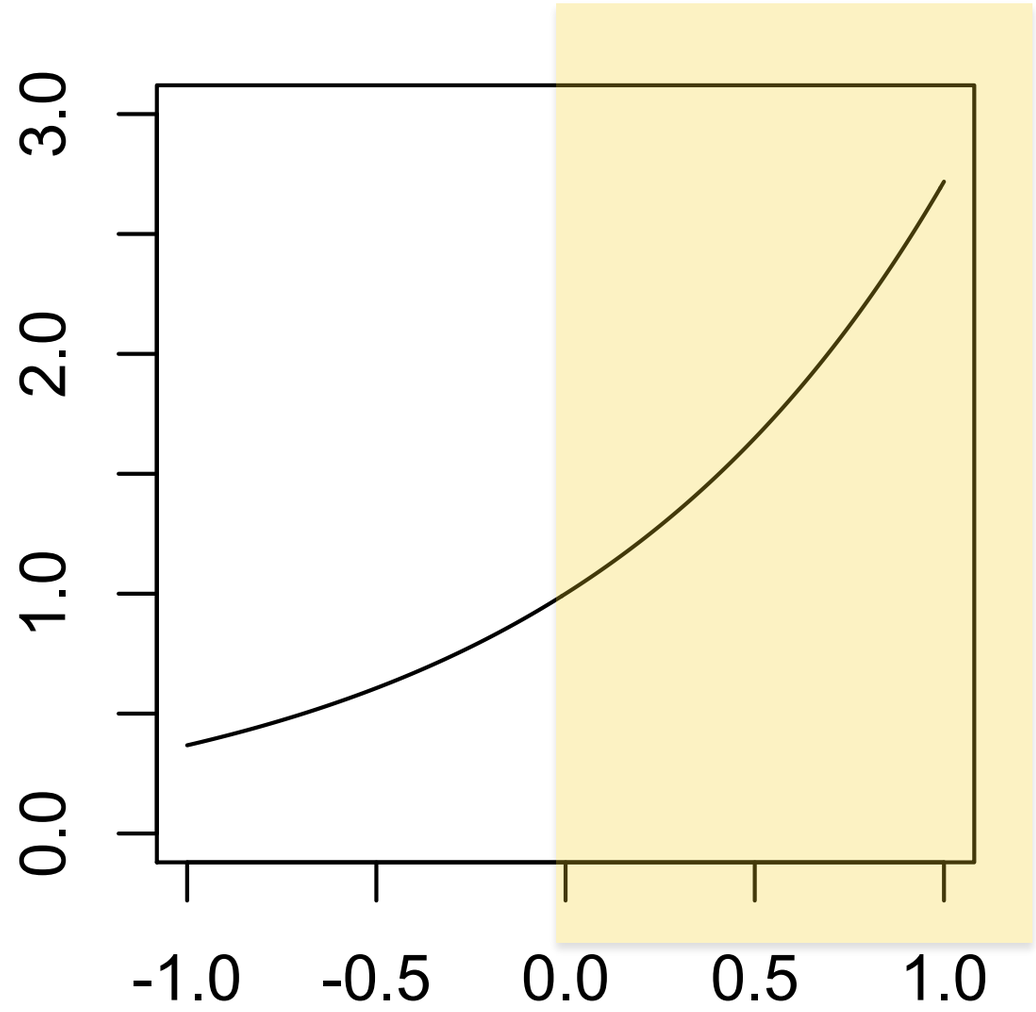
P($F(s)$,$F(s_{new})$, T)

(1) **if** $F(s_{new}) > F(s)$ **then return** 1.0
(2) **else return** $e^{\frac{F(s_{new})-F(s)}{T}}$

Acceptance Probability

$$P = e^{\frac{F(s_{new}) - F(s)}{T}}$$

- Borrowed from metallurgy
- When new solution is better ($F(s_{new}) > F(s)$), always accept ($P > 1$)
- When new solution is equally good, accept
- When new solution is worse:
  - more likely to accept small downhill movement
  - gets smaller as temperature drops



Cooling Schedule

Temperature at time step t as a function of t:

- Linear: $T(t) = T_0 - \alpha t$
- Exponential: $T(t) = T_0 \alpha^t (0 < \alpha < 1)$
- Logarithmic $T(t) = \frac{c}{\log(t+d)}$

  - With large c, this can be very slow cooling
  - There is an existence proof that says logarithmic will find the global optimum in infinite time… huh?
  - It becomes essentially a random search
  - Theoretically interesting, but practically not so much.

Tabu Search

- Another attempt to escape local optima
- Two exceptions to local search:
  - It is possible to accept a worse move
  - Remember “visited” solutions and avoid coming back

Tabu Search

TabuSearch()
(1) $s \leftarrow s_0$
(2) $s_{best} \leftarrow s$
(3) $T \leftarrow []$ // tabu list
(4) **while** not stoppingCondition()
(5) $c_{best} \leftarrow null$
(6) **foreach** $c \in$ GetNeighbours(s)
(7) **if** $(c \notin T) \wedge (F(c) > F(c_{best}))$ **then** $c_{best} \leftarrow c$
(8) $s \leftarrow c_{best}$
(9) **if** $F(c_{best}) > F(s_{best})$ **then** $s_{best} \leftarrow c_{best}$
(10) append(T, c_{best})
(11) **if** $|T| > maxTabuSize$ **then** removeAt$(T, 0)$
(12) **return** sBest

Tabu list is a FIFO queue: with the **`maxTabuSize`** we can control the memory span of the search.

Random Restart

- Search budget is usually given in limited time ("terminate after 5 minutes") or in number of fitness evaluation ("terminate after 5000 fitness evaluations")

- If a local search reaches optima and budget remains? Start again from another random solution and keep the best answer across multiple runs.

Search Radius

- For local search algorithms to be effective: the search space may be large, but the search radius should be reasonably small

- Search radius: the number of moves required to go **across** the search space

Search Radius: TSP

- Travelling Salesman Problem: what is the shortest path that visits all N cities?

- Search Space: N! (e.g 2,432,902,008,176,640,000 when N = 20)

- Search Radius: at most N(N-1)/2 swaps to change any permutation of cities to any other (e.g. 190 when N=20)

Summary

- Local search: direct use of fitness landscape concept, with various mechanism to escape local optima.

- Easy to implement, easy to understand what is going on; good for insights into landscape

- Design of search space (especially discrete one) affects the performance of search